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Project 3.2

Problem 9.71: Car Battery Lifetimes

Part (a) - Constructing the 95% Confidence Interval for Variance

We are given a sample of battery lifetimes: 1.9, 2.4, 3.0, 3.5, and 4.2 years. The manufacturer claims that the variance is 1 year². We are tasked with constructing a 95% confidence interval for the variance and determining if the manufacturer's claim is valid.

The formula for the chi-square confidence interval for variance 𝜎^2is:

Where:

n=5 (sample size)

is the sample variance

=0.05 (since it’s a 95% confidence interval)

are the chi-square critical values with 𝑛 -1 =4 degrees of freedom.

We first calculate the sample variance

Sample mean

Sum of squared deviations:

Sample variance:

The chi-square critical values for 4 degrees of freedom are:

Using these values, we can calculate the confidence interval for

Conclusion:

The 95% confidence interval for the variance is (0.346, 7.98). Since the manufacturer's claim that =1 is within this interval, we do not have sufficient evidence to reject the claim.

Problem 16.3: Foreign Impurities in Jam

Part (a) - Sign Test at the 0.05 Significance Level

We are given the impurity percentages for 16 jars of jam: 2.4, 2.3, 3.1, 2.2, 2.3, 1.2, 1.0, 2.4, 1.7, 1.1, 4.2, 1.9, 1.7, 3.6, 1.6, and 2.3 percent. We are tasked with performing a sign test to determine whether the median impurity percentage is 2.5% at the 0.05 significance level.

1. Formulate Hypotheses:

Null hypothesis (): The median percent of impurities is 2.5%.

Alternative hypothesis (): The median percent of impurities is not 2.5%.

1. Sign Test Procedure:

We first compare each data point to the hypothesized median of 2.5%:

Values greater than 2.5%: 3.1, 4.2, 3.6

Values less than 2.5%: 2.4, 2.3, 2.2, 2.3, 1.2, 1.0, 2.4, 1.7, 1.1, 1.9, 1.7, 1.6, 2.3

Number of positive signs (values greater than 2.5%): 3

Number of negative signs (values less than 2.5%): 13

1. Normal Approximation to the Binomial Distribution:

Since we have more than 10 data points, we can use the normal approximation to the binomial distribution. The test statistic is calculated as:

Where:

X=3 (number of positive signs)

𝑛=16 (total number of jars)

Calculating the z-value:

1. Decision Rule:

For a two-tailed test at the 0.05 significance level, the critical z-values are ±1.96. If the calculated z-value is less than -1.96 or greater than 1.96, we reject the null hypothesis.

1. Conclusion:

Since 𝑧=−2.5 is less than -1.96, we reject the null hypothesis. There is enough evidence to conclude that the median percent of impurities in this brand of jam is not 2.5%.